

PREPARATORY GUIDE PHYSICS

FOR
CAMBRIDGE A LEVEL
UTME
POST UTME

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CHAPTER TWO

KINEMATICS

Definition of terms

Displacement: It is the distance moved in a particular direction. It is a vector quantity. The S.I unit is (m)

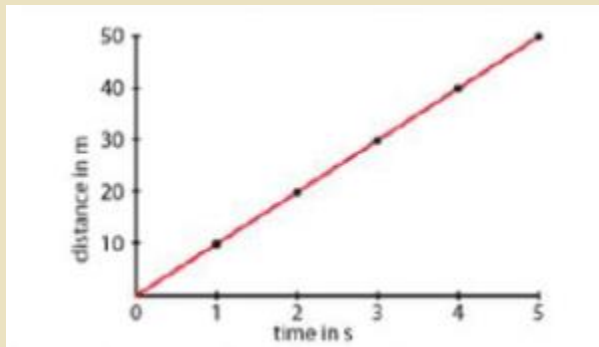
Velocity: It is the rate of change of displacement. It is a vector quantity. The S.I unit is (ms^{-1})

Speed: It is the rate of change of distance. It is a scalar quantity. The S.I unit is (ms^{-1})

Acceleration: It is the rate of change of velocity. It is a vector quantity. The S.I unit is (ms^{-2})

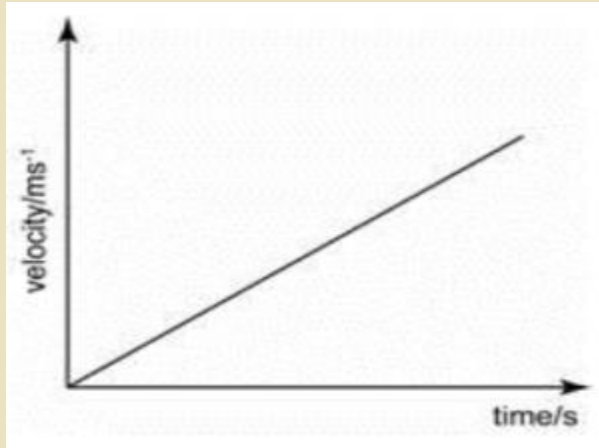
Uniform Speed: This is equal distance at equal time interval

Graphical representation of Uniform Speed



Uniform Acceleration: This is equal velocity at equal time interval

Graphical representation of uniform acceleration



Mathematical Expressions

Speed = distance / time = d / t

Velocity = displacement / time = d / t

Average speed: Total distance / total time taken

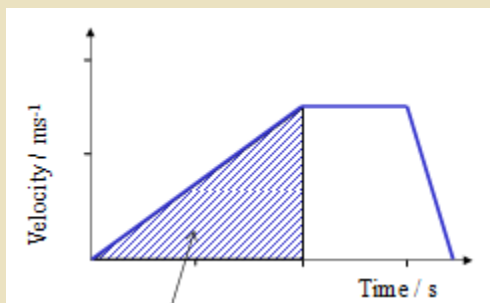
Acceleration = change of velocity / time = $(v - u) / t$

v = Final velocity

u = initial velocity

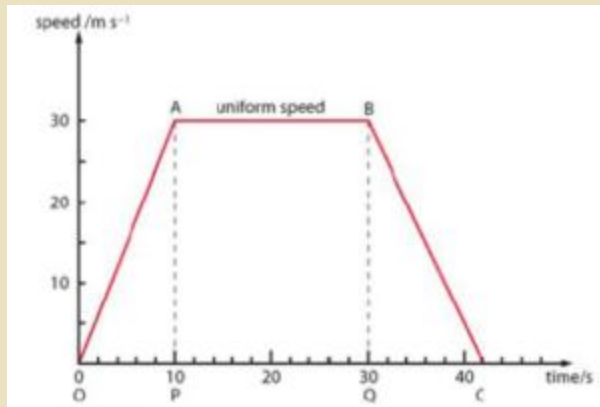
Velocity – time Graph

The area under the graph of a velocity – time graph is displacement



Since the diagram is a trapezium, we can calculate the total distance using the area of a trapezium

Let look at an example



From the diagram above

- (i) What is the distance travelled during the first 10s
- (ii) The total distance travelled
- (iii) The average speed for the whole journey

Solution

i Using triangle AOP = $\frac{1}{2} \times OP \times AP = \frac{1}{2} \times b \times h = \frac{1}{2} \times 10 \times 30 = 150\text{m}$

ii Using Trapezium AOCBA = $\frac{1}{2} \times (AB + OC) \times AP = \frac{1}{2} \times (20 + 42) \times 30 = \frac{1}{2} \times 62 \times 30 = 930\text{m}$

ii Average speed = total distance / total time taken

Total time taken is 42s

Average speed = $930 / 42 = 22.1 \text{ ms}^{-1}$

Derive, from the definitions of velocity and acceleration, equations that represent uniformly accelerated motion in a straight line

$$\mathbf{v = u + at}$$

$$\mathbf{s = \frac{u+v}{2} * t}$$

$$\mathbf{v^2 = u^2 + 2as}$$

$$\mathbf{s = ut + \frac{1}{2}at^2}$$

v = final velocity

u = initial velocity

t = time taken

s = distance covered

a = uniform acceleration

From the definition of acceleration

$$a = \frac{v-u}{t}$$

Cross multiply and rearrange

$$v = u + at$$

From average velocity,

$$\text{Average velocity} = s / t$$

$$\text{Average velocity} = (v + u) / 2$$

Equate the two equations together, you get

$$s / t = (v + u) / 2$$

Cross multiply and rearrange

$$s = [(v + u)t] / 2$$

Taking $v = u + at$ i , and $s = [(v + u)t] / 2$ii

Substiturte $v = u + at$ in eqn i into eqn ii, so you get,

$$s = ut + 1/2at^2$$

Taking $v = u + at$i and $s = [(v - u)t] / 2$ ii

Make t the subject of the eqn in eqn i

$$t = (v - u) / a \text{iii}$$

substitute eqn iii into eqn ii, you get

$$v^2 = u^2 + 2as$$

Motion of bodies falling in a uniform gravitational field without air resistance

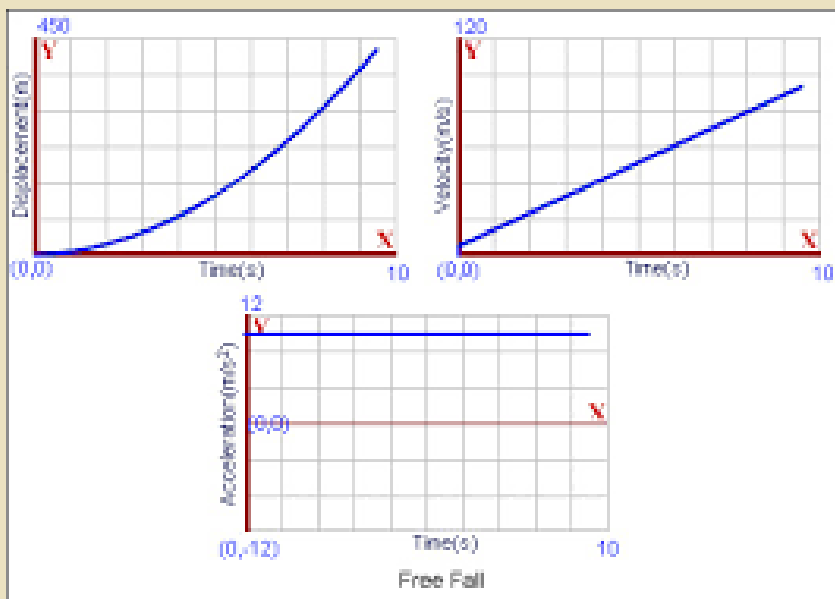


Image from xtremepapers.com

In a uniform gravitational field, the gravitational field strength is constant. The gravitational field strength is also referred to as acceleration due to gravity (g).

Without air resistance means the air resistance is negligible or the drag force

The displacement – time graph, velocity – time graph, acceleration – time graph of such a motion is shown below



Always remember that the gradient or slope of displacement – time graph is velocity, the gradient of velocity-time graph is acceleration.

From the graph above, the acceleration-time graph shows that it is a uniform acceleration or a constant acceleration.

In this case all you have to do is make $a = 9.81 \text{ ms}^{-2}$, which is the value of acceleration due to gravity. But in some cases you can be asked to make $a = 10 \text{ ms}^{-2}$

The motion of bodies falling in a uniform gravitational field with air resistance

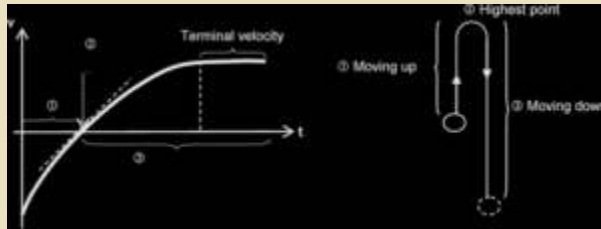


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When any object moves through air, the air offers a frictional resistance (drag) to the motion. This causes the object to decelerate. The deceleration is not constant but depends on the velocity of the object.

Graphical representation of this kind of motion is shown below

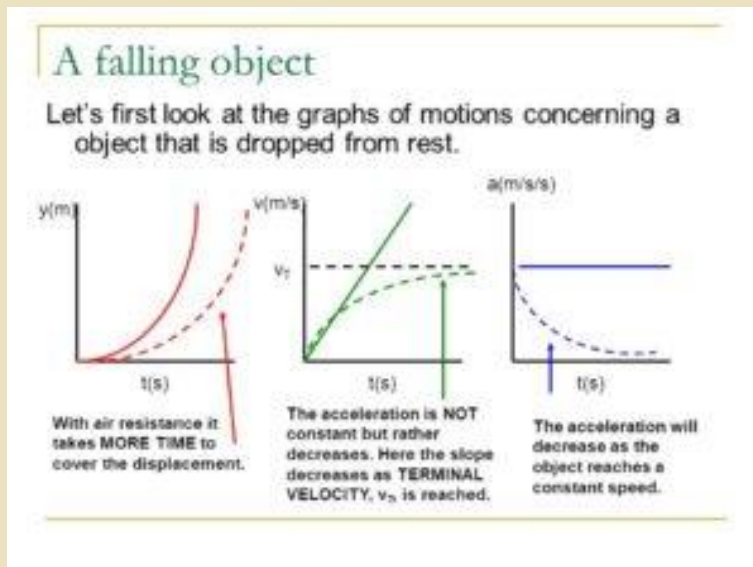


Image from slideplayer.com

Where the arrow pointed shows the graph of displacement-time graph, velocity-time graph, acceleration-time graph when there is air resistance.

For displacement-time graph: it takes more time to cover because of the air resistance compared with when there is no air resistance

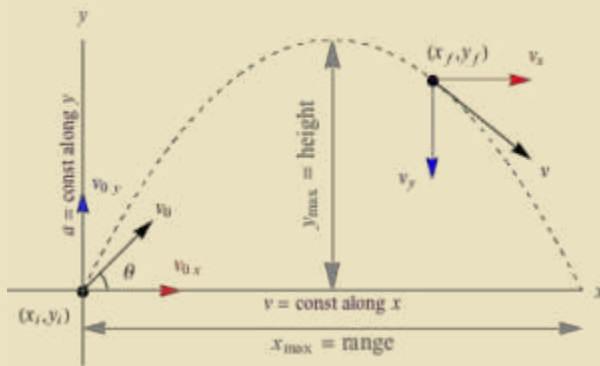
For velocity: the acceleration isn't constant because as the speed increases the air resistance increases until a time is reached when the weight of the object equals the drag force (air resistance), at this point no resultant force acting on

the body and it will fall with a constant speed, called the terminal velocity (this has being explained when i discussed dynamics)

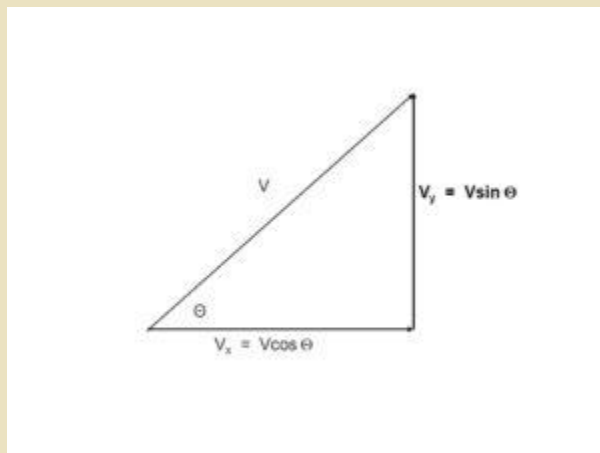
Terminal velocity is the point at which the resultant force is zero.

Motion due to a uniform velocity in one direction and a uniform acceleration in a perpendicular direction.

The type of motion to discuss here is a Projectile. Projectile is any object that is given an initial velocity and then follows a path determined entirely by gravitational acceleration.



The initial velocity is at an angle of θ . This initial velocity will be resolve into both vertical component and horizontal component.



Using your SOHCAHTOA

The vertical component of the velocity will be $u \sin \theta$

The horizontal component of the velocity will be $u \cos \theta$

Horizontal component of the velocity has no force acting so it is constant

Vertical component of the velocity has a constant force acting so there is a constant acceleration.

Derivations

To calculate the time to reach the maximum height

$$v = u - at$$

At maximum height $v = 0$

$$0 = u \sin \theta - at$$

$$t = u \sin \theta / a$$

$$a = 9.81 \text{ ms}^{-2}$$

$$\text{Time of flight} = 2 * t = 2u \sin \theta / a$$

To calculate maximum height

$$v^2 = u^2 - 2as$$

At maximum height $v = 0$

$$u = u \sin \theta$$

$$a = 9.81 \text{ ms}^{-2}$$

$s =$ maximum height

If you substitute you get your maximum height

Horizontal distance (range)

$$v = d / t$$

The reason for this formula is because there is no force acting on it, which implies a uniform velocity

$d =$ horizontal distance

$$v = \text{horizontal component of velocity} = u \cos \theta$$

t = time to complete the parabolic path (which can sometimes time of flight or time to reach the maximum height depending on the projectile diagram given). If it is a full diagram you use time of flight. If it is half of the full diagram you use time to reach the maximum height). The diagram above is a full diagram.

$$u \cos \theta = d / t$$

Worked Examples

Question 1

A ball is thrown vertically down towards the ground with an initial velocity of 4.23 m s^{-1} . The

ball falls for a time of 1.51 s before hitting the ground. Air resistance is negligible.

(a) (i) Show that the downwards velocity of the ball when it hits the ground is 19.0 m s^{-1} .

(ii) Calculate, to three significant figures, the distance the ball falls to the ground.

(b) The ball makes contact with the ground for 12.5 ms and rebounds with an upwards

velocity of 18.6 m s^{-1} . The mass of the ball is 46.5 g .

(i) Calculate the average force acting on the ball on impact with the ground

Solution

$$v = u + at$$

$$u = 4.23 \text{ ms}^{-1}$$

$$t = 1.51 \text{ s}$$

$$a = 9.81$$

$$v = 4.23 + 1.51 * 9.81$$

$$v = 19.04 \text{ ms}^{-1}$$

ii

$$s = ut + \frac{1}{2}at^2$$

$$s = 4.23 \times 1.51 + \frac{1}{2} \times 9.81 \times 1.51^2$$

$$s = 17.57 \text{ m}$$

b

$$f = \frac{m(v - u)}{t}$$

Since ball moves in opposite direction after the rebound

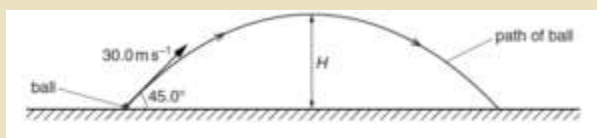
$$f = \frac{m(v + u)}{t}$$

$$f = \frac{0.0465(19 + 18.6)}{0.0125}$$

$$F = 140 \text{ N}$$

Question 2

A ball of mass 400 g is thrown with an initial velocity of 30.0 m s^{-1} at an angle of 45.0° to the horizontal, as shown in fig below (Cambridge past question may / june 2014 p22 q4)



Air resistance is negligible. The ball reaches a maximum height H after a time of 2.16 s.

(i) Calculate

1. the initial kinetic energy of the ball,
2. the maximum height H of the ball

Solution

$$E_k = \frac{1}{2} m v^2$$

$$m = 400\text{g} = 0.4\text{kg}$$

$$v = 30\text{ms}^{-1}$$

$$E_k = \frac{1}{2} \times 0.4 \times 30^2 = 180 \text{ J}$$

To calculate the maximum

$$v^2 = u^2 - 2as$$

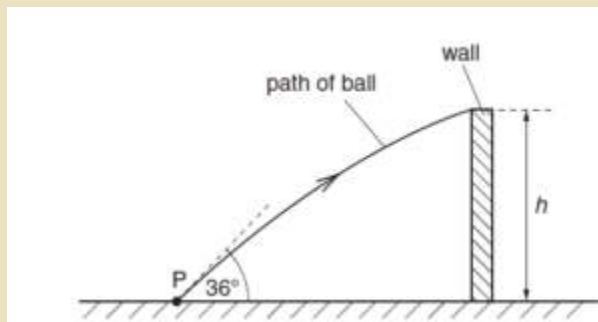
$$v = 0$$

u is the vertical component of the velocity = $u \sin \theta = 30 \sin 45 = 21.21 \text{ ms}^{-1}$

$$s = \frac{21.21^2}{2 \times 9.81} = 22.94\text{m}$$

Question 3

A ball is thrown from a point P, which is at ground level, as illustrated in figure below



The initial velocity of the ball is 12.4 m s^{-1} at an angle of 36° to the horizontal. The ball just passes over a wall of height h . The ball reaches the wall 0.17 s after it has been thrown. (Cambridge past question oct / nov 2010 p22 que 2)

Assuming air resistance to be negligible, calculate

- (i) the horizontal distance of point P from the wall,
- (ii) the height h of the wall.

Solution

The horizontal distance is

$$v = d/t$$

$$v = u \cos \theta = 12.4 \cos 36 = 10.03 \text{ ms}^{-1}$$

$$d = vt = 10.03 * 0.17 = 1.7 \text{ m}$$

The height h

$$s = ut - \frac{1}{2} a t^2$$

The u here is the vertical component of the initial velocity $u \sin \theta = 12.4 \sin 36 = 7.29 \text{ ms}^{-1}$

$$s = 7.29 * 0.17 - \frac{1}{2} * 9.81 * 0.17^2 = 1.24 - 0.14 = 1.1 \text{ m}$$

Question 4

An object is projected from a height of 80m above the ground with a velocity of 40 ms^{-1} at an angle of 30° to the horizontal. The time of flight is

A. 16s B. 10s C. 8s D. 4s

[$g = 10 \text{ ms}^{-2}$]

Solution

$$T = \frac{2u \sin \theta}{g}$$

$$T = \frac{2 * 40 * \sin 30}{10}$$

$$T = 40 / 10$$

$$T = 4 \text{ s}$$